Correlation in Estimation of the Q value*

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Abstract Q value, the ratio of charmless hadronic decays between ψ' and J/ψ, is evaluated to be $(26.0 \pm 3.5)\%$. The dependence in evaluation is taken into account carefully, and several approaches are adopted to find out the correlation coefficient or covariance between different measurements.

Key words Q value, correlation, J/ψ and ψ' decay

New estimation estimated in 2009.2.20

1 Introduction

From the perturbative QCD (pQCD), it is expected that both J/ψ and ψ' decaying into light hadrons are dominated by the annihilation of $c\bar{c}$ into three gluons, with widths proportional to the square of the wave function at the origin [1]. This yields the pQCD "12% rule", that is

$$Q_h \quad = \quad \frac{\mathcal{B}_{\psi' \to h}}{\mathcal{B}_{J/\psi \to h}} = \frac{\mathcal{B}_{\psi' \to e^+e^-}}{\mathcal{B}_{J/\psi \to e^+e^-}} = (12.3 \pm 0.7)\% \quad . \tag{1}$$

The violation of the above rule was first observed in $\rho\pi$ and $K^{*+}K^- + c.c.$ modes by Mark II [2], since then BES has measured many two-body decay modes of ψ' which violate this rule [3]. There are some modes which are suppressed in ψ' decays relative to 12% rule, like Vecter-Pseudoscalar (VP) and Vecter-Tensor (VT) decay modes; while there are others which are enhanced, like $K^0_S K^0_L$ mode. There have been many theoretical efforts trying to explain the puzzle [4], however, none explains all the existing experimental data satisfactorily and naturally.

Some phenomenological studies indicate [5, 6, 7] that the S- and D-wave charmonia mixing scenario provides us an unified model to explain both the suppressed and the enhanced decays of ψ'. If the scenario of S- and D-wave mixing is correct, it should give correct relation between the branching fraction of J/ψ, ψ' and ψ'' decays to the same mode. Therefore by virtue of measurements of J/ψ and ψ', we can estimate the corresponding decay width of ψ'' to the same final states.

A recent research [7] tries to utilize the available information from J/ψ and ψ' decays to evaluate the corresponding charmless decay width of ψ''. For such study, it is crucial to know the proportion and extent of the suppressed and enhanced ψ' decays. A major impediment to doing estimation in a systematic manner is the dearth of ψ' branching fraction measurements. So an alternative approach, whose main idea is to obtain charmless decay proportion by subtracting the charmed decays, is employed to estimate Q_h. In the light of the approach, a value Q_g equivalent to Q_h, which is defined as

$$Q_g = \frac{\mathcal{B}(\psi' \to ggg + \gamma gg)}{\mathcal{B}(J/\psi \to ggg + \gamma gg)} \quad , \tag{2}$$

has been estimated based on experimental results to be $Q^1_g = (23 \pm 7)$ % in Ref. [8] and $Q^2_g = (24.0 \pm 5.6)$ % in Ref. [9].

It is clear that such Q_g value is considerably enhanced with respect to Q_h, let alone that of suppressed decay modes. Such estimation implicates that while some modes are suppressed in ψ' decays, a substantial

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fraction of ψ' decay modes is enhanced relative to J/ψ, with the average enhancement factor of more than 2.

It is also worthy of noticing that the difference between two previous Q_g's. Although Q_g^1 and Q_g^2 are estimated on the basis of the same values from PDG2002 [10], their central value differ by 5%, moreover, the relative error of these two estimations differ from each other by about 30%. Such large discrepancy should be understood.

As Q_g is important in understanding of charmonium decay, and the discrepancy between previous estimations is rather prominent, it is necessary to evaluate Q_g again based on the new PDG [11].

2 Evaluation of Q value

The estimation of Q_g is based on an assumption that the decays of J/ψ and ψ' in the lowest order of QCD are classified into hadronic decays via three gluons (ggg), electromagnetic decays via virtue photon (γ^*), radiative decays into light hadrons via two gluons (γgg), and decays to lower mass charmonium states ($c\bar{c}X$) [9, 12]. Thus, using the relation $\mathcal{B}(ggg)+\mathcal{B}(\gamma gg)+\mathcal{B}(\gamma^*)+\mathcal{B}(c\bar{c}X)=1$, one can derive $\mathcal{B}(ggg)+\mathcal{B}(\gamma gg)$ by subtracting $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\bar{c}X)$ from unity.

The calculated values of $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\bar{c}X)$, together with the values used to calculate them are summarized in Table 1. As regards to ψ', two faint branching fractions of final states $\gamma\eta(2S)$ and $h_c(^1P_1)+X$ are neglected in our evaluation. Herein special attention should be paid to the dependence between various branching fractions. In fact, many approaches have been used to acquire the correlation coefficient or covariance between different measurements.

Table 1: Experimental data on branching fractions for J/ψ and ψ' decays through virtual photon and to lower mass charmonium states used in our analysis. Most of data are taken from PDG [11], except for $\mathcal{B}(J/\psi,\psi'\to\gamma^*\to$ hadrons), which calculated by product $R\cdot\mathcal{B}(J/\psi,\psi'\to\mu^+\mu^-)$, where $R=2.28\pm0.04$ [13]. In estimating the errors of the sums, the correlations between the channels are considered meticulously.

Channel	$\mathcal{B}(J/\psi)$	$\mathcal{B}(\psi')$	ord. of ψ'
$\gamma^*\to$ hadrons	(13.4±0.33)%	(1.66±0.18)%	⑩
e^+e^-	(5.93±0.10)%	$(7.55\pm0.31)\times10^{-3}$	⑦
$\mu^+\mu^-$	(5.88±0.10)%	$(7.3\pm0.8)\times10^{-3}$	⑧
$\tau^+\tau^-$		$(2.8\pm0.7)\times10^{-3}$	⑨
$\gamma^*\to X$	(25.22±0.43)%	(3.43±0.27)%	
$\gamma\eta_c$	(1.3 ±0.4)%	$(2.8\pm0.6)\times10^{-3}$	⑪
$\pi^+\pi^-J/\psi$		(31.7±1.1)%	①
$\pi^0\pi^0J/\psi$		(18.8±1.2)%	②
$\eta J/\psi$		(3.16±0.22)%	③
π^0J/ψ		$(9.6\pm2.1)\times10^{-4}$	⑫
$\gamma\chi_{c0}$		(8.6±0.7)%	④
$\gamma\chi_{c1}$		(8.4±0.8)%	⑤
$\gamma\chi_{c2}$		(6.4±0.6)%	⑥
$c\bar{c}X$	(1.3±0.4)%	(77.4±2.5)%	

2.1 Estimation involving J/ψ decay

In order to evaluate the charmless hadronic decay of J/ψ, we need the branching ratios $\mathcal{B}_e$, $\mathcal{B}_\mu$, $\mathcal{B}_{\gamma^*}$ and $\mathcal{B}_{\gamma\eta_c}$, and the correlation information involving these channels. Since the experiment for $\gamma\eta_c$ is unrelated with other experiments [11], so there is no correlation between $\mathcal{B}_{\gamma\eta_c}$ and other branching ratios. As to other correlations, we will consider them one by one.

The leptonic branching ratios of J/ψ decay listed in Table 1 are actually the synthetic results from four experiment groups, whose measurements, according to PDG, are presented in Table 2. These results

can classified into two categories: one from energy scan experiment, the other from the analysis of $\psi' \to \pi^+\pi^- J/\psi,\ J/\psi \to \ell^+\ell^-$.

Table 2: Experimental data on branching fractions for J/ψ and ψ' decays to lepton pair. Correlation coefficients between measurement of e^+e^- and $\mu^+\mu^-$ states are also listed.

ord.	Experiment	Collab.	$\mathcal{B}_e(\%)$	$\mathcal{B}_\mu(\%)$	Γ_e/Γ_μ	Coeff. (ρ)
1	e^+e^- scan	BES [14]	6.09 ± 0.33	6.08 ± 0.33	1.00 ± 0.07	0.167
2	e^+e^- scan	MARKI [15]	6.9 ± 0.9	6.9 ± 0.9	1.00 ± 0.05	0.927
3	$\pi^+\pi^- J/\psi$	BES [16]	$5.90 \pm 0.05 \pm 0.10$	$5.84 \pm 0.06 \pm 0.10$		0.764
4	$\pi^+\pi^- J/\psi$	MARKIII [17]	$5.92 \pm 0.15 \pm 0.20$	$5.90 \pm 0.15 \pm 0.19$		0.629
5	$\pi^+\pi^- J/\psi$	CLEO [18]	$5.945 \pm 0.067 \pm 0.042$	$5.960 \pm 0.065 \pm 0.050$	0.997 ± 0.014	0.534

As to the scan experiment, let's consider the relation

$$R_{e\mu} = \frac{\Gamma_e}{\Gamma_\mu} = \frac{\mathcal{B}_e}{\mathcal{B}_\mu} \quad . \tag{3}$$

By the virtue of error propagation formula, we have

$$\nu^2_{R_{e\mu}} = \nu_e^2 + \nu_\mu^2 - 2\rho_{e\mu} \cdot \nu_e \cdot \nu_\mu \quad ,$$

or

$$\rho_{e\mu} = \frac{\nu_e^2 + \nu_\mu^2 - \nu_R^2}{2\nu_e \nu_\mu} \quad , \tag{4}$$

where $\nu_{R_{e\mu}},\ \nu_e,\ \nu_\mu$ indicate the relative error for $R_{e\mu},\ \mathcal{B}_e,\ \mathcal{B}_\mu$ respectively. Since $\nu_{R_{e\mu}},\ \nu_e,\ \nu_\mu$ are readily obtained from values in Table 2, the correlation coefficients can be calculated by Eq. (4), and are listed in the last column in Table 2.

As to the second kind of results, which were presented with statistic and systematic uncertainties, we notice that the systematic uncertainty is common for e^+e^- and $\mu^+\mu^-$ final states analysis. By virtue of the theory purposed in Ref. [19], if two measurements x_i and x_j, have the common normalized uncertainty σ_f, and their measurements are reported as follows

$$\begin{aligned} x_i^{rep.} &= x_i(1 \pm \sigma_f) \pm \sigma_i \quad , \\ x_j^{rep.} &= x_j(1 \pm \sigma_f) \pm \sigma_j \quad , \end{aligned}$$

where σ_i and σ_j are the uncommon uncertainties, then the correlation coefficient between x_i and x_j can be calculated by

$$\rho_{ij} = \frac{\sigma_f^2 x_i x_j}{\sqrt{\sigma_i^2 + (x_i\sigma_f)^2} \cdot \sqrt{\sigma_j^2 + (x_j\sigma_f)^2}} \quad . \tag{5}$$

In our case, σ_i and σ_j are statistic uncertainty and σ_f is 1.7% for BES [16] and 3.3% for MARKII [17].

In the following analysis, we directly use e^2 and μ^2 (short for σ_e^2 and σ_μ^2) to represent the covariance of branching ratio $\mathcal{B}_e$ and $\mathcal{B}_\mu$. We know the reported results in PDG are the weighted average of several experiments, viz.

$$\overline{\mathcal{B}}_e = \sum_{i=1}^{5} a_i \mathcal{B}_{ei} \quad , \qquad \overline{\mathcal{B}}_\mu = \sum_{i=1}^{5} b_i \mathcal{B}_{\mu i} \quad , \tag{6}$$

where

$$\begin{aligned} a_i &= \frac{\overline{e}^2}{e_i^2} \ , \quad \overline{e}^2 = 1 \Big/ \sum_{i=1}^{5} \frac{1}{e_i^2} \ , \\ b_i &= \frac{\overline{\mu}^2}{\mu_i^2} \ , \quad \overline{\mu}^2 = 1 \Big/ \sum_{i=1}^{5} \frac{1}{\mu_i^2} \ . \end{aligned} \tag{7}$$

with $i(=1,2,3,4,5)$ denotes the four groups of experiment measurements in Table 2. Then the transformation relation between $\overline{\mathcal{B}}_e$ ($\overline{\mathcal{B}}_\mu$) and $\mathcal{B}_{ei}$ ($\mathcal{B}_{\mu i}$) is expressed as

$$\left(\begin{array}{c} \overline{\mathcal{B}}_e \\ \overline{\mathcal{B}}_\mu \end{array} \right) = \mathbf{SB} \ , \tag{8}$$

with

$$\mathbf{B}^T = (\ \mathcal{B}_{e1} \quad \mathcal{B}_{e2} \quad \mathcal{B}_{e3} \quad \mathcal{B}_{e4} \quad \mathcal{B}_{e5} \quad \mathcal{B}_{\mu 1} \quad \mathcal{B}_{\mu 2} \quad \mathcal{B}_{\mu 3} \quad \mathcal{B}_{\mu 4} \quad \mathcal{B}_{\mu 5}\ )\ ,$$

here superscript T denotes the transposition of vector or matrix and

$$\mathbf{S} = \left(\begin{array}{cccccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \right) \ .$$

Using the relation in Eq. (8), the covariance matrix $\overline{\mathbf{V}}$ for $\overline{\mathcal{B}}_e$ and $\overline{\mathcal{B}}_\mu$ can be formulated as [20]

$$\overline{\mathbf{V}} = \mathbf{SVS}^T \ , \tag{9}$$

where $\mathbf{V}$ is the covariance matrix for $\mathcal{B}_{ei}$ and $\mathcal{B}_{\mu i}$, which has the following form

$$\mathbf{V} = \left(\begin{array}{cccccccc} e_1^2 & & & & \rho_1 e_1 \mu_1 & & & \\ & e_2^2 & & & & \rho_2 e_2 \mu_2 & & \\ & & e_3^2 & & & & \rho_3 e_3 \mu_3 & \\ & & & e_4^2 & & & & \rho_4 e_4 \mu_4 \\ \rho_1 e_1 \mu_1 & & & & \mu_1^2 & & & \\ & \rho_2 e_2 \mu_2 & & & & \mu_1^2 & & \\ & & \rho_1 e_1 \mu_1 & & & & \mu_1^2 & \\ & & & \rho_2 e_2 \mu_2 & & & & \mu_1^2 \end{array} \right) \ . \tag{10}$$

Here naught elements have been suppressed and ρ_i indicates four correlated coefficients listed in Table 2. Notice the definition of $a,\ b$ in Eq. (7), we obtain

$$\overline{\mathbf{V}} = \left(\begin{array}{cc} \overline{e}^2 & \overline{e}^2 \overline{\mu}^2 \cdot \sum_{i=1}^{5} \frac{\rho_i}{e_i \mu_i} \\ \overline{e}^2 \overline{\mu}^2 \cdot \sum_{i=1}^{5} \frac{\rho_i}{e_i \mu_i} & \overline{\mu}^2 \end{array} \right) = \left(\begin{array}{cc} 3.71 & 2.32 \\ 2.32 & 4.07 \end{array} \right) \times 10^{-3} \ , \tag{11}$$

together with the definition of $\overline{\mathcal{B}}_e$ and $\overline{\mathcal{B}}_\mu$ in Eq. (6), we work out the synthetic results

$$\begin{array}{rcl} \overline{\mathcal{B}}_e & = & (5.939 \pm 0.061)\% \ , \\ \overline{\mathcal{B}}_\mu & = & (5.931 \pm 0.064)\% \ , \end{array}$$

which are just the values given by PDG (refer to Table 1). At the same time, we also acquire the correlation between $\overline{\mathcal{B}}_e$ and $\overline{\mathcal{B}}_\mu$ as reflecting by $\overline{\mathbf{V}}$ in Eq. (11).

Next we consider the correlation between $\mathcal{B}_{\gamma^*}$ and other branching ratios. As we mentioned in Table 1, we calculate the $\mathcal{B}_{\gamma^*}$ by the product

$$\mathcal{B}_{\gamma^*} = R \cdot \mathcal{B}_\mu \ , \tag{12}$$

therefore $\mathcal{B}_{\gamma^*}$ is correlated with $\mathcal{B}_\mu$ and with it alone. In order to figure out the correlation coefficient, we consider the relation

$$R = \frac{\mathcal{B}_{\gamma^*}}{\mathcal{B}_\mu} \ , \tag{13}$$

which is similar to that of Eq. (3), so the corresponding Eq. (5) can be utilized to work out the coefficient $\rho_{\mu\gamma^*}$ between $\mathcal{B}_\mu$ and $\mathcal{B}_{\gamma^*}$, which is 0.524 for J/ψ decays and 0.987 for ψ' decays.

Summarizing the fore analyses, we obtain the covariance for J/ψ estimation

$$\mathbf{V}_{J/\psi} = \begin{array}{c} \\ \mathcal{B}_e \\ \mathcal{B}_\mu \\ \mathcal{B}_{\gamma^*} \\ \mathcal{B}_{\gamma\eta_c} \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{B}_e & \mathcal{B}_\mu & \mathcal{B}_{\gamma^*} & \mathcal{B}_{\gamma\eta_c} \end{array} \\ \left(\begin{array}{cccc} \sigma_e^2 & \rho_{e\mu}\sigma_e\sigma_\mu & 0 & 0 \\ \rho_{e\mu}\sigma_e\sigma_\mu & \sigma_\mu^2 & \rho_{\mu\gamma^*}\sigma_\mu\sigma_{\gamma^*} & 0 \\ 0 & \rho_{\mu\gamma^*}\sigma_\mu\sigma_{\gamma^*} & \sigma_{\gamma^*}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\gamma\eta_c}^2 \end{array} \right) \end{array} \ . \tag{14}$$

With above covariance matrix, we work out $\mathcal{B}(J/\psi \to ggg) + \mathcal{B}(J/\psi \to \gamma gg) = (73.48 \pm 0.59)\%$.

2.2 Estimation involving ψ' decay

Since 2002, the treatment of the branching ratios of the ψ' and $\chi_{c0,1,2}$ has undergone an important restructuring.

When measuring a branching ratio experimentally, it is not always possible to normalize the number of events observed in the corresponding decay mode to the total number of particles produced. Therefore, the experimenters sometimes report the number of observed decays with respect to another particle in the relevant decay chain. This is actually equivalent to measuring combinations of braching frations of several decay modes. To extract the branching ratio of a given decay mode, the collaborations use some previously reported measurements of the required branching ratios. However, the values are frequently taken from the *Review of Particle Physics (RPP)*, which in turn uses the branching ratio reported by the experiment in the following edition, giving rise either to correlations or plain vicious circles. The way to avoid various dependencies and correlations is to extract the branching ratios through a fit that uses the truly measured combinations of branching fractions and partial widths. This fit, in fact, should involve decays from the four concerned particles, ψ', χ_{c0}, χ_{c1}, and χ_{c2}, and occasionally some combinations of branching ratios of more than one of them. This is what is done since the 2002 edition [10]. According to such a global fit of the new PDG [11], we can acquire the correlation coefficients for most of channels (channels from ① to ⑨) listed in Table 1, only the correlation coefficients for channels ⑩, ⑪, and ⑫ have to be considered separately.

Similar to the analysis of previous section, $\mathcal{B}(⑩)$ (i.e. $\mathcal{B}_{\gamma^*}$) is only correlated with process $\mathcal{B}_\mu$ and its correlated coefficient has been given in the previous section. As to $\mathcal{B}(⑪)$ and $\mathcal{B}(⑫)$, their correlations with other quantities are neglected for their comparatively small absolute values.

So based on our analysis and with the information from PDG, we obtain the correlation coefficients for ψ' estimation

$$\mathbf{C}_{\psi'} = \begin{array}{cc} & \begin{array}{cccccccccccc} \text{①} & \text{②} & \text{③} & \text{④} & \text{⑤} & \text{⑥} & \text{⑦} & \text{⑧} & \text{⑨} & \text{⑩} & \text{⑪} & \text{⑫} \end{array} \\ \begin{array}{c} \text{①} \\ \text{②} \\ \text{③} \\ \text{④} \\ \text{⑤} \\ \text{⑥} \\ \text{⑦} \\ \text{⑧} \\ \text{⑨} \\ \text{⑩} \\ \text{⑪} \\ \text{⑫} \end{array} & \left(\begin{array}{cccccccccccc} 1 & & & & & & & & & & & \\ 0.48 & 1 & & & & & & & & & & \\ 0.14 & 0.13 & 1 & & & & & & & & & \\ 0.15 & 0.07 & 0.02 & 1 & & & & & & & & \\ 0.02 & 0.01 & 0 & 0 & 1 & & & & & & & \\ 0.05 & 0.03 & 0.01 & 0.01 & 0 & 1 & & & & & & \\ 0.69 & 0.69 & 0.21 & 0.10 & 0.02 & 0.05 & 1 & & & & & \\ 0.30 & 0.17 & 0.05 & 0.04 & 0.01 & 0.02 & 0.22 & 1 & & & & \\ 0.15 & 0.07 & 0.02 & 0.02 & 0 & 0.01 & 0.10 & 0.04 & 1 & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.98 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \end{array} \quad . \tag{15}$$

Notice the symmetry of covariance matrix, so the symmetric elements are omitted in above matrix. The total covariance can be calculated by

$$\sigma^2_{\psi'} = \sum_{i=\text{①}}^{\text{⑫}} \sum_{j=\text{①}}^{\text{⑫}} \rho_{ij}\sigma_i\sigma_j \quad , \tag{16}$$

where the values of ρ_{ij} are presented in Eq. (15). With $\sigma^2_{\psi'}$, we work out $\mathcal{B}(\psi' \to ggg) + \mathcal{B}(\psi' \to \gamma gg) = (19.14 \pm 2.54)\%$.

2.3 Q value

Using the results of previous sections, the ratio of branching fractions of ψ' to J/ψ decays into hadrons is given by

$$Q_g = \frac{\mathcal{B}(\psi' \to ggg + \gamma gg)}{\mathcal{B}(J/\psi \to ggg + \gamma gg)} = (26.0 \pm 3.5)\% \quad . \tag{17}$$

The central value of above estimation is consistent with the forementioned estimations Q^1_g [8] and Q^2_g [9], but with higher accuracy.

The relation between the decay rates of ggg and γgg is readily calculated in pQCD to the first order as [21]

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}\left(1 - 2.9\frac{\alpha_s(m_c)}{\pi}\right) \quad . \tag{18}$$

Using $\alpha_s(m_c) = 0.28$, one can estimate the ratio to be 0.062. A similar relation can be deduced for the ψ' decays. If we adopt 6.2% as esimation for ratio $\mathcal{B}(\gamma gg)/\mathcal{B}(ggg)$, we obtain $\mathcal{B}(J/\psi \to ggg) \simeq (69.2 \pm 0.6)\%$ and $\mathcal{B}(\psi' \to ggg) \simeq (18.0 \pm 2.4)\%$, while the "26.0% ratio" stands well for either ggg or γgg. Although Q_g is considerably enhanced relative to Q_h, it coincides with the ratio for $K^0_S K^0_L$ decay between ψ' and J/ψ, which according to recent results from BES [22, 23] is

$$Q_{K^0_S K^0_L} = (28.8 \pm 3.7)\% \ . \tag{19}$$

The relative higher Q_g implicates that a substantial fraction of ψ' decay modes is enhanced relative to J/ψ, or some modes which exist in ψ' decays, are absent in J/ψ decays.

In order to understand the discrepancy of relative error between Q^1_g and Q^2_g, we also preform the corresponding calculation without consideration of any possible correlation. First, we work out $\mathcal{B}(J/\psi \to ggg) + \mathcal{B}(J/\psi \to \gamma gg) = (73.48 \pm 0.54)\%$ and $\mathcal{B}(\psi' \to ggg) + \mathcal{B}(\psi' \to \gamma gg) = (19.14 \pm 2.05)\%$, respectively, whose relative error is 10% and 20% smaller than those of dependently estimated results. Then using Eq. (17), we obtain $Q_g = (26.0 \pm 2.8)\%$, comparing with $Q_g = (26.0 \pm 3.5)\%$, the difference between two relative error is up to 25%, which is just at the same level of the discrepancy for previous estimations[1]. Because there is no detailed exposition about Q value estimation, we have no idea about what kinds of and to what extent the correlation has been considered in previous estimation, and we can merely point out that the different treatment of correlation may be the reason leading to the discrepancy of Q value estimation between Refs. [9] and [8].

3 Discussion

It is worth to mention another approach for estimating Q_h [9], which is to use the data on branching fractions for hadronic decays in final states containing pions, kaons, and protons that have already been measured for both J/ψ and ψ'. They are $\pi^+\pi^-$, K^+K^-, $p\overline{p}$, $\pi^+\pi^-\pi^0$, $p\overline{p}\pi^0$, $2(\pi^+\pi^-)$, $3(\pi^+\pi^-)$, $2(\pi^+\pi^-)\pi^0$, $3(\pi^+\pi^-)\pi^0$, $\pi^+\pi^-K^+K^-$, $\pi^+\pi^-p\overline{p}$. Using the PDG data compiled in Table 3, we have

$$\sum_{i=1}^{11} \mathcal{B}(J/\psi \to f_i) = (10.87 \pm 0.74)\%$$

and

$$\sum_{i=1}^{11} \mathcal{B}(\psi' \to f_i) = (1.01 \pm 0.19)\% \ .$$

It follows that

$$\begin{aligned} Q_s &= \sum_{i=1}^{11} \mathcal{B}(\psi' \to f_i) \Big/ \sum_{i=1}^{11} \mathcal{B}(J/\psi \to f_i) \\ &= (9.30 \pm 1.82)\% \ . \end{aligned} \tag{20}$$

We notice ddthat the Q_s not only differ from Q_e greatly but is suppressed with respect to Q_h as well.

A remark is in order here. We know that most of multihadron final states in fact include sums of several two-body intermediate states. One thus observes a mixed effect which may deviate noticeably from the expected value of Q, even if a few of the two-body intermediates are severely suppressed. For example, the decay $\psi' \to \pi^+\pi^-K^+K^-$ can proceed through intermediate state $K^*(892)^0\overline{K}^*_2(1430)^0 + c.c.$,

[1] As forementioned, the correlations of $\mathcal{B}$(①) and $\mathcal{B}$(⑫) with other quantities are neglected. Otherwise, if we assume that one of, or all of these two branching ratios are completely correlated with other quantities, we can obtain the following results:

$$\begin{aligned} \mathcal{B}(1)\ \text{correlated}: &\quad \Rightarrow (19.14 \pm 2.654)\% \quad \Rightarrow (26.0 \pm 3.62)\%; \\ \mathcal{B}(12)\ \text{correlated}: &\quad \Rightarrow (19.14 \pm 2.579)\% \quad \Rightarrow (26.0 \pm 3.51)\%; \\ \text{both}\mathcal{B}(1)\ \text{and}\mathcal{B}(12)\ \text{correlated}: &\quad \Rightarrow (19.14 \pm 2.693)\% \quad \Rightarrow (26.0 \pm 3.67)\% \ . \end{aligned}$$

Table 3: Branching fractions for ψ' and J/ψ decays [11], and Q_h values are also calculated.

Modes	Channels	$\mathcal{B}_{J/\psi}(10^{-3})$	$\mathcal{B}_{\psi(2S)}(10^{-4})$	Q_h (%)
0^-0^-	$\pi^+\pi^-$	0.147 ± 0.023	0.8 ± 0.5	54 ± 35
	K^+K^-	0.237 ± 0.031	1.0 ± 0.7	42 ± 30
$B\overline{B}$	$p\overline{p}$	2.12 ± 0.10	2.07 ± 0.31	9.8 ± 1.5
$3\ 0^-$	$\pi^+\pi^-\pi^0$	$21.2 \pm 1.01\dagger$	0.8 ± 0.5	0.53 ± 0.34
$0^-B\overline{B}$	$\pi^0 p\overline{p}$	1.09 ± 0.09	1.4 ± 0.5	12.8 ± 4.7
multibody decay	$2(\pi^+\pi^-)$	4.0 ± 1.0	4.5 ± 1.0	11.3 ± 3.8
	$3(\pi^+\pi^-)$	4.0 ± 2.0	1.5 ± 1.0	3.7 ± 3.1
	$3(\pi^+\pi^-)\pi^0$	29 ± 6	35 ± 16	12.1 ± 6.1
	$2(\pi^+\pi^-)\pi^0$	33.7 ± 2.6	30 ± 8	8.9 ± 2.5
	$\pi^+\pi^-K^+K^-$	7.2 ± 2.3	16 ± 4	22.2 ± 9.0
	$\pi^+\pi^-p\overline{p}$	6.0 ± 0.5	8.0 ± 2.0	13.3 ± 3.5

†: The weighted average of two new measurements, one from BES [24] $(2.10 \pm 0.12)\%$ and BABAR [25] $(2.18 \pm 0.19)\%$.

whose $Q = 2.9 \pm 1.3$, is greatly suppressed [26]. So the Q_s is not exact ratio of ψ' to J/ψ inclusive hadronic decay rates, but represents on average the ratio of the exclusive decay channels, as measured to date.

In fact, the current results concerning the ψ' decays is rather limited, even sum all charmless channels presented in PDG2004 [11], the total branching fraction is less 2%. Such condition prevents us from accurately testing the Q_h value. However, the estimation of Q value provides us some clues concerning the exploration of charmonium decay dynamics. Since many suppressed channels have been found, especially those such as $\rho\pi$ which is greatly suppressed in ψ' decay, and if the Q_h is really represent the averaged value of inclusive hadronic decay, the estimation of Q_g indicates that either lots of enhanced decays are not discovered, or some particular decays only present at ψ', or both cases exist. Therefore, systematic study of ψ' decays are anxiously awaited.

4 Summary

Estimation of Q value, the ratio of charmless hadronic decays between J/ψ and ψ', is very important to understand and explore the dynamics of charmonium decay. In our paper, the dependence in evaluation is taken into account carefully, and several approaches are adopted to find out the correlation coefficient or covariance between different measurements. The final evaluation of Q value is $(26.0 \pm 3.5)\%$, whose accuracy is much better than prevoius estimations.

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References

[1] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34**, 43 (1975);
A. De Rújula and S. L. Glashow,Phys. Rev. Lett. **34**, 46 (1975).

[2] M. E. B. Franklin *et al.*, Phys. Rev. Lett. **51**, 963 (1983).

[3] C. Z. Yuan, "Hadronic decays of charmonia from BESII", talk at MESON 2004: 8th International Workshop of Meson Production, Properties and Interactions, Cracow, Poland, 4-8, June 2004 (hep-ex/0407024); X. H. Mo, "Recent BES Results on $\psi(2S)$ decay", talk at DAFNE 2004: Workshop on Physics at Meson Factories, Frascati, Italy, 7-11, June 2004(hep-ex/0408031).

[4] W. S. Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983); G. Karl and W. Roberts, Phys. Lett. **144 B**, 263 (1984); S. J. Brodsky *et al.*, Phys. Rev. Lett. **59**, 621 (1997); M. Chaichian and N. A. Törnqvist, Nucl. Phys. B **323**, 75 (1989); S. S. Pinsky, Phys. Lett. B **236**, 479 (1990); X. Q. Li, D. V. Bugg and B. S. Zou, Phys. Rev. D **55**, 1421 (1997); S. J. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 4682 (1997); Y. Q. Chen and E. Braaten, Phys. Rev. Lett. **80**, 5060 (1998); J. M. Gérard and J. Weyers, Phys. Lett. B **462**, 324 (1999); T. Feldmann and P. Kroll, Phys. Rev. D **62**, 074006 (2000); J. P. Ma, Phys. Rev. D **65**, 097506 (2002); M. Suzuki, Phys. Rev. D **65**, 097507 (2002).

[5] P. Wang, X. H. Mo and C. Z. Yuan, Phys. Rev. **D 70**, 077505 (2004).

[6] J. L. Rosner, Phys. Rev. **D 64**, 094002 (2001).

[7] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Rev. **D 70**, 114014 (2004).

[8] M. Suzuki, Phys. Rev. **D 63**, 054021 (2001).

[9] Y. F. Gu and X. H. Li, Phys. Rev. D **63**, 114019 (2001).

[10] K. Hagiwara *et al.*,(Particle Data Group), Phys. Rev. D **66**, 010001 (2002).

[11] S. Eidelman *et al.*, (Particle Data Group), Phys. Lett. B **592**, 1 (2004).

[12] L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).

[13] K. K. Seth, Phys. Rev. D **69**, 097503 (2001).

[14] J. Z. Bai *et al.*, Phys. Lett. B **355**, 374 (1995).

[15] A. M. Boyarski *et al.*, Phys. Rev. Lett. **34**, 1357 (1975).

[16] J. Z. Bai *et al.*, Phys. Rev. D **58**, 092006 (1998).

[17] D. M. Coffman *et al.*, Phys. Rev. Lett. **68**, 282 (1992).

[18] Z. Li *et al.*, Phys. Rev. D **71**, 111103 (2005).

[19] G. D'Agostini, Nucl. Instr. Meth. **A 346**, 306-311 (1994).

[20] S. Brandt, Data Analysis, Springer-Verlag, New York Inc. 1999 (third edition).

[21] W. Kwong, Phys. Rev. D **37**, 3210 (1988).

[22] J. Z. Bai *et al.*, Phys. Rev. D **69**, 012003 (2004).

[23] J. Z. Bai *et al.*, Phys. Rev. Lett. **92**, 052001 (2004).

[24] J. Z. Bai *et al.*, Phys. Rev. D **70**, 012005 (2004).

[25] B. Aubert *et al.*, Phys. Rev. D **70**, 072004 (2004).

[26] J. Z. Bai *et al.*, Phys. Rev. D **69**, 072001 (2004).